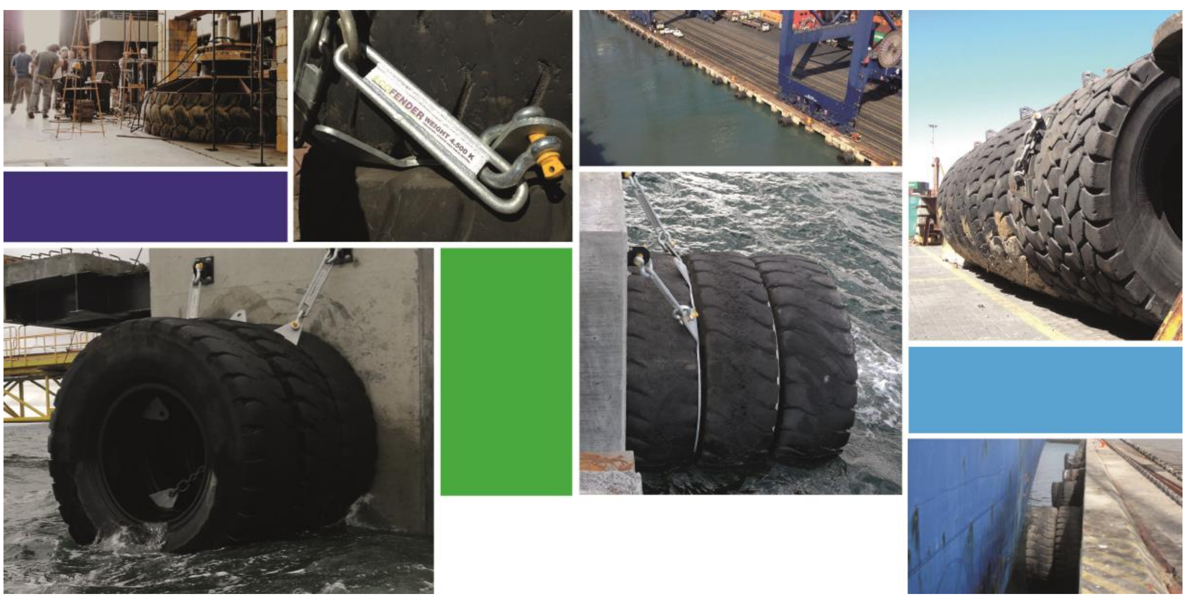
MAGELLAN
CONSULTING GROUP
MARINE SOLUTIONS

MEMORY OF CALCULATION

DOCK FENDERS

ECOFENDER-2300



Miguel Lagos Infante, PhD
Professor, Faculty of Engineering
University of Talca, Chile

(+569) 6251 6487
mlagos@ecofender.cl

ECO FENDER
DOCK FENDERS

Ecofender 2300. Structural parts

1. Anchoring to dock side

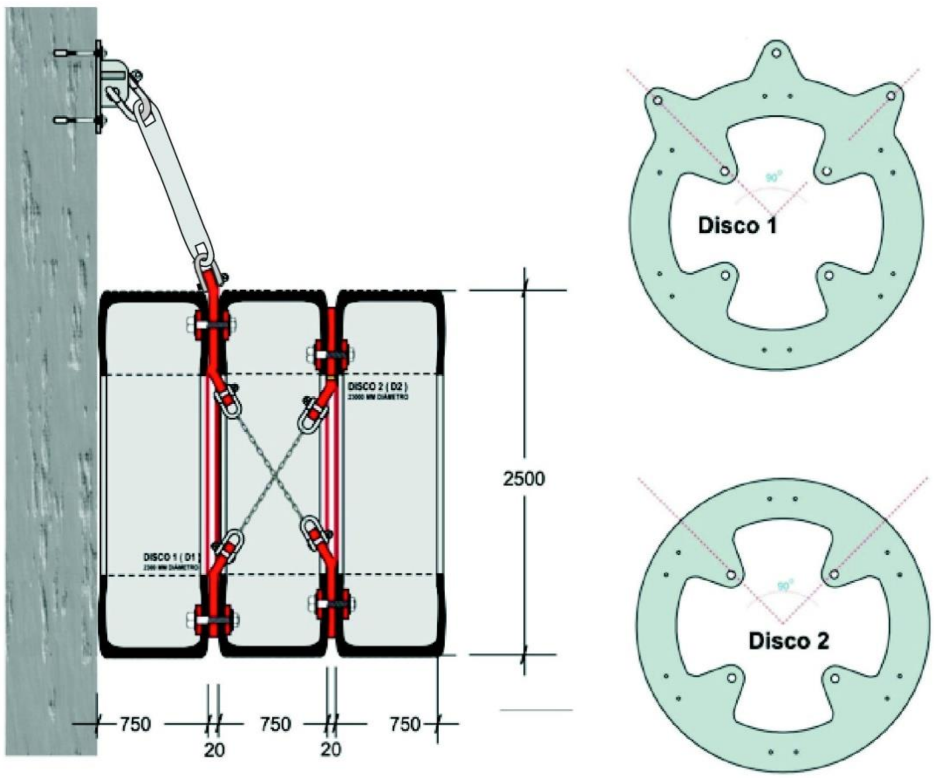


Figure 1

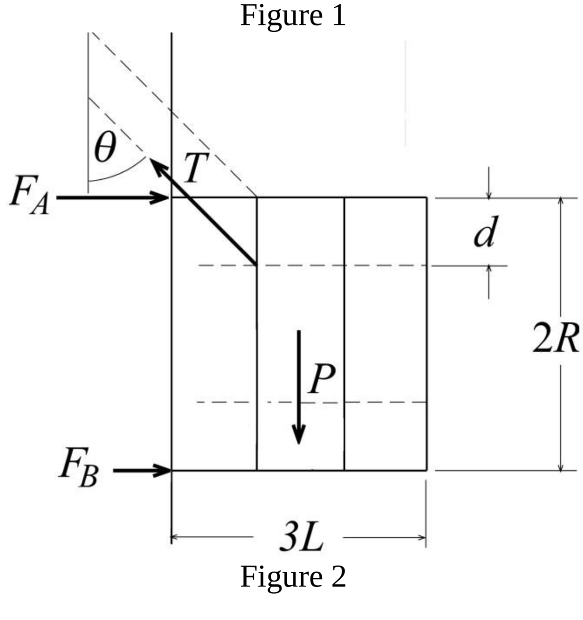


Figure 2

(+569) 6251 6487
mlagos@ecofender.cl

Berthing of a vessel may demand reaction forces which may reach several hundreds of kN per dock fender (1kN=1000 Newton), however these forces are normally not transmitted to the parts fastening the Ecofender fenders to the wharf front face. If the vessel approaches the dock perpendicularly to the dock side, pressing through the fender the dock with a force orthogonal to its front face, the process does not increases the tension of the fixing chains. However, the vessel may press the fender making a displacement with a non-vanishing component in the direction parallel to the surface of the wharf front face, besides to the normal component. The parallel components may be vertical by effect of the sea waves or horizontal by, for example, the elasticity of the berthing ropes. In this situation the forces may, in principle, be transmitted to the fixing elements, which are not rigid but are constituted by chains that support the weight of the fender. Nevertheless this can hardly occur because before the rubber fender will slide over the concrete of the wharf front face, sliding of the steel surface of the vessel side on the opposite surface of the fender should take place.

The friction coefficient between rubber and the steel surface may vary between 0.6 and 0.85, and the most probable value is 0.7. The friction coefficient of tire rubber over concrete has been extensively studied by its importance in vehicles, and all studies give values over 0.9. The most frequent values are between 0.9 and 1.1, but can reach 1.39 (D. Ammon, R. Gnadler, G. Mäckle, H. J. Unrau, ATZ Worlwide, vol. 106, No. 7−8, pp 34−38, 2004). (Friction coefficient between solid surfaces cannot be greater than unity, but it can when elastomers are involved).

In conclusion, the forces over the fixing elements of the Ecofender fenders come principally from the fender weight. The forces exerted by the vessel are not transmitted to the elements fixing the fender to the wharf front face. Ecofender fenders have a fairly rigid structure and take advantage of friction forces with the dock face.

Mechanical stability of mounting

When not working, the fender hangs from the chains and rests on the wharf vertical face. The first point is to verify is the stability of the structure under the action of the forces in the idle situation. The problem of Figure 1 is hiperestatic because the central anchoring, represented by the upper chain, and the lateral anchoring, represented by the lower chains are both able to produce equilibrium by themselves. The load on the two systems can be chosen by adjusting the tension of the chains. Thus, to safely dimension the anchoring parts it is convenient to assuume each of the two systems performing the whole task. Figure 2 shows the forces operating on the sysytem in the idle condition. If $d = 0$ the scheme of Figure 2 depicts the central anchoring, assuming a positive value for distance d then Figure 2 represents the lateral anchoring.

In rigor, the force exerted by the wharf side vertical face on the fender is more or less homogeneouly distributed, however one can replace the distributed forces by the two localized forces F_A and F_B. Force T may represent the tension of the central chain (setting $d = 0$) or the resultant of the tensions of the two lateral chains (setting $d > 0$). The equilibrium equations lead to

(+569) 6251 6487
mlagos@ecofender.cl

$$T = \frac{P}{\cos\theta}$$

$$F_A = \left[-\frac{L}{4R} + \left(1 - \frac{d}{2R}\right)\tan\theta\right]P$$

$$F_B = \left(\frac{L}{4R} + \frac{d}{2R}\tan\theta\right)P\ ,$$

where $d = 0$ describes the central anchoring working alone, and $d > 0$ corresponds to the lateral anchoring with no help of the central one.

Forces F_A and F_B must be both non negative, that is, must have the sense indicated in Figure 2, because the wharf front face can only press the fender over all its surface, and cannot pull from it. Force F_B is always non negative and the condition for $F_A > 0$ is

$$\tan\theta > \frac{L}{2(2R - d)}.$$

Then, setting $L = 0.75$ m and $R = 1.25$ m, the situation is stable for $\theta > 8.5°$ with $d = 0$, and $\theta > 16.7°$ for the extreme value $d = R$. In conclusion, the stability of the fender mounting is ensured for a small value of angle θ.

Weigth of fender and tension of the mounting chains

The mass of each tire is stimated as $M_1 = 1200$ kg. Each disk of 20 mm thick steel has an external diameter of 2.200 m and an internal one of 1.600 m, and hence its mass without the triangular elements in the external perimeter is $M_2 = 281$ kg (density $\rho = 7850\ \mathrm{kg/m^3}$). The mass of each triangular element of 0.020 m thick steel (sides 0.040, 0.040 and 0.050 m) is estimated as $M_3 = 12{,}3$ kg. The mass of the whole fender is then

$$3M_1 + 2M_2 + 11M_3 = 4297\ \mathrm{kg}\,.$$

In practice the fenders accumulate water. Drilling drain holes in the tread is unsafe and reduces significantly the capacity for energy storage. A tire of external radius R_2, internal radius R_1 and width L accumulates a maximal water mass

$$M_4 = \rho L R_2^2\left[\cos^{-1}\left(\frac{R_1}{R_2}\right) - \frac{R_1}{R_2}\sqrt{1 - \left(\frac{R_1}{R_2}\right)^2}\right],$$

where $\rho = 1.025\ \mathrm{ton/m^3}$ is the density of sea water. Setting $R_2 = 1.15$ m, $R_1 = 0.62$ m and $L = 0.75$ m, it turns out to be $M_4 = 556\ \mathrm{kg}$. The weight of a complete fender, with mass $3 \times M_4$ of cumulated water can be taken as

$$P = 5965\ \text{kg-f} = 58.46\ \mathrm{kN}\,.$$

(a) Central anchoring

The angle of the chain with respect to vertical is $\theta = 45°$ and the corresponding tension is

(+569) 6251 6487
mlagos@ecofender.cl

$$T = \frac{P}{\cos\theta} = 8436 \text{ kg-f} = 82.67 \text{ kN}.$$

(b) Lateral anchoring at 90°

The resultant of the tensions of the two chains in a plane at 45° from the vertical direction is 8436 kg-f. Denoting T' the tension of each chain, the resultant is $T = 2T' \cos 45°$ and

$$T' = 5965 \text{ kg-f}.$$

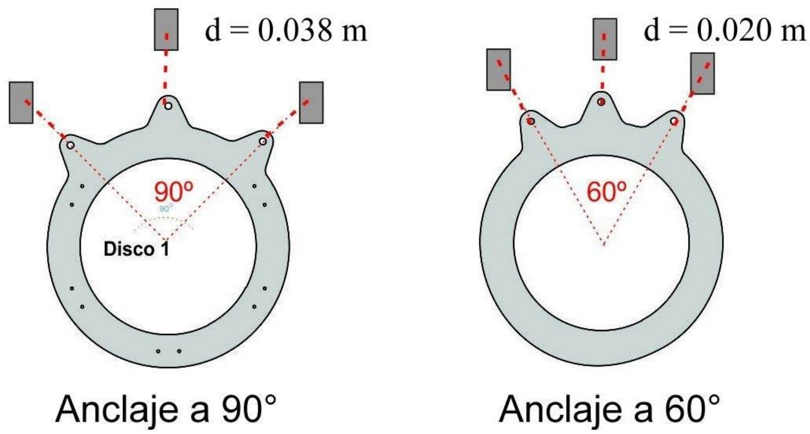


Figure 3

(c) Lateral anchoring at 60°

The resultant of the tensile forces T' of the two chains satisfies $T = 2T' \cos 30°$ and then

$$T' = 4870 \text{ kg-f}.$$

Dimensioning chains and anchoring bolts

The most demanding situation occurs when the central anchoring makes the whole work. The chain tension is in this case $T = 8436$ kg-f $= 82.67$ kN. Assuming a safety factor 1.2 the chain mus be selected to resist a load of 10123 kg-f = 99.21 kN.

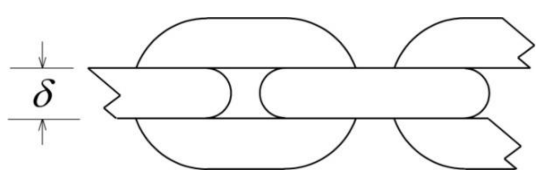


Figure 4

Alternatively, considering that the yield stress σ_Y for the chain steel is that of the A36 steel ($\sigma_Y = 250$ MPa), it must be

$$\sigma = \frac{T}{2\pi\delta^2/4} < \sigma_Y ,$$

where δ is the diameter of the ring bars. From this relation we have

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mlagos@ecofender.cl

$$\delta > \sqrt{\frac{2 \times 99210}{\pi \times 250 \times 10^6}} = 0.0159 \text{ m} \quad \text{(diameter of ring bars)}.$$

Then the diameter of the ring bars must greater than 1.6 cm = 0.63". The recommended chain is the one characterized by number 3338 in the list included below, of high resistance chains grade 80 in norm ISO 3076.

75226	Metro / Metro / Tira: 100 mts	16mm	48mm	Terminación: Negra Carga de trabajo: 8.00 Ton. Carga de ruptura: 32.0 Ton. Certificación: Shenli Rigging Co. Peso: 5.63kg/mt		
3338	Metro / Metro / Tira: 80 mts	19mm	56mm	Terminación: Pulida Carga de trabajo: 12.8Ton Carga de ruptura: 51.2Ton Peso: 8.7kg/mt		
6899	Metro / Metro / Tira: 40 mts	19mm	56,1mm	Terminación: Negra Carga de trabajo: 11.3Ton Carga de ruptura: 46.5Ton Peso: 8.7kg/mt		

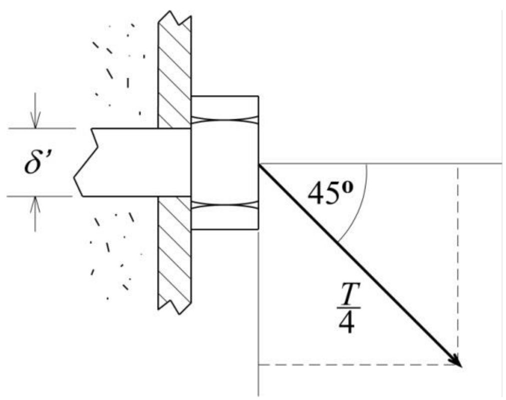

Figure 5

Chains are fixed to the wharf front face by a steel ring welded to a plate fastened to the concrete by four anchoring bolts. The anchoring bolts of the central chain, which is the one supporting the largest stress when doing the main work, must support forces making an angle of 45° with respect to the bolts axes, as seen in Figure 5. The bolt is subjected to combined normal and shear stresses, σ and τ. The normal stress comes from the sum of the force due to the torque applied to bolts when mounting the system and the normal component $T \cos 45°/4$ of the tension of the chain applied to each of the four bolts. The shear stress is determined only by the other component T sen $45°/4$ of the force exerted by the chain.

To determine the normal load due to the mounting torque it is assumed that the steel plate must exert a moderate pressure over the concrete surface when the chain is working with a tension $T = 10123$ kg-f $= 99.21$ kN. This is ensured by producing a load slightly

(+569) 6251 6487
mlagos@ecofender.cl

greater than $T\cos 45°/4$ by effect of the mounting torque given to the bolts. This way, the force of the chain over the plate almost equilibrates the force exerted by the bolt heads over the plate. The difference between both forces presses the concrete without producing a considerable maximal friction force, and thus the shear forces on the bolts are the ones preventing the plate sliding over the concrete surface. The normal load exerted on a bolt is then $2T$ sen 45° and the stresses are

$$\sigma = \frac{1}{4}\frac{2T\cos 45°}{\pi\delta'^2/4}, \qquad \tau = \frac{1}{4}\frac{T\sin 45°}{\pi\delta'^2/4}.$$

The bolt initiates deformation when semi-difference between the principal stresses becomes greater than the flow stress (Apendix A). Thus the condition for the mechanical stability of the system is

$$\sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} < \frac{\sigma_Y}{2}.$$

Combining these equations and replacing $\sin 45° = \cos 45° = 1/\sqrt{2}$ it turns out that the diameter δ' of the bolts must be such that

$$\delta' > \sqrt{\frac{2T}{\pi\sigma_Y}}.$$

Replacing the tension 99.21 kN, which incorporates a safety factor, and assuming $\sigma_Y = 36\text{ ksi} = 250\text{ MPa}$, which corresponds to a grade 2 bolt, a modest bolt quality, it is obtained that

$$\delta' > 0.0159\text{ m} = 1.6\text{ cm} = 0.63\text{"} \text{ (minimal diameter for mounting bolts).}$$

Alternatively, bolt choice can be done demanding that can support a load greater than

$$F = \frac{\pi\delta'^2}{4}\sigma_Y = 49.64\text{ kN} = 5065\text{ kg-f.}$$

In practice anchoring bolts of 3/4 x 8” are used, which is a considerably oversized dimension according to the previous calculations.

Eye plate and pad plate, and shackle

Figure 6 shows the eye plate anchoring the fender to the wharf vertical front face, as well as the shakle and part of the chain. Each fender is supported by three of these systems, one placed directly above and two at both sides. The latter two are located in such a way that the corresponding chains make angles of 30° (or 45°) with the vertical in the elevation or frontal view, and an angle of 45° in the lateral view.

Figure 7 indicates the forces operating on the anchoring parts when the chain is subjected to a tension T. It is assumed that the chain may have an angular deviation θ with respect to the vertical direction in the frontal view. The calculations below asume the hypothesis that the pad plate undergoes a plane elastic deformation, thus the normal stresses σ, which are proportional to the unitary deformations at each point, follow a linear law of the general form

(+569) 6251 6487
mlagos@ecofender.cl

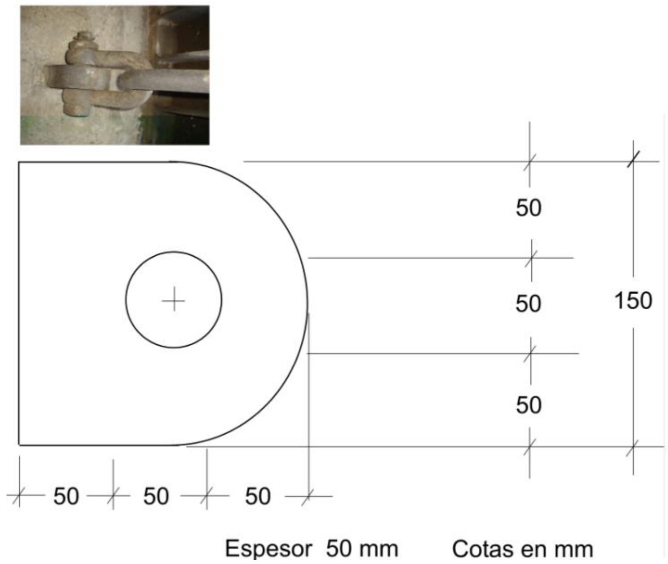

Figure 6

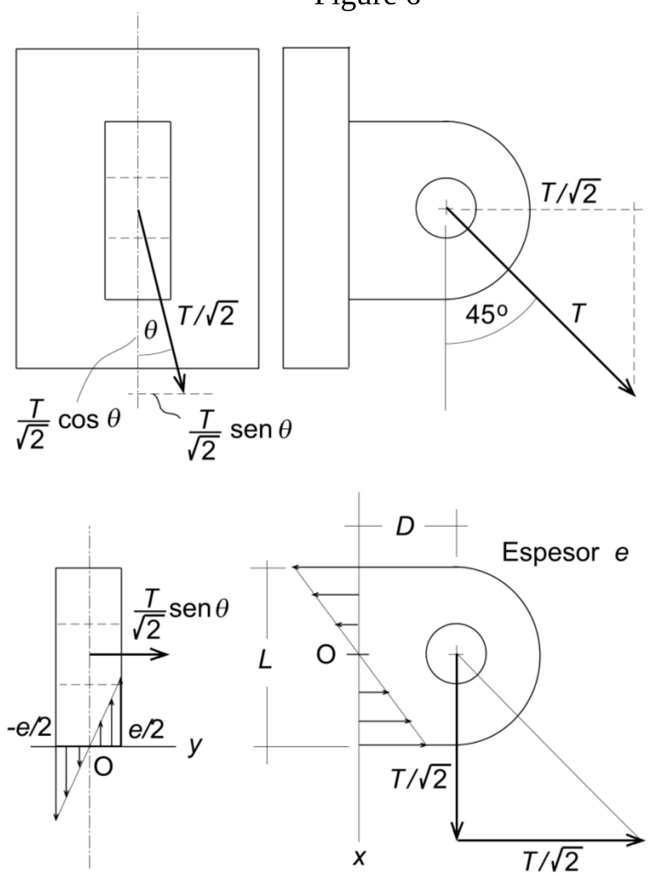

Figure 7

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mlagos@ecofender.cl

$$\sigma(x,y)=\frac{T}{\sqrt{2}Le}+\alpha x+\beta y,\qquad -\frac{L}{2}\leq x\leq\frac{L}{2}\,,\qquad -\frac{e}{2}\leq y\leq\frac{e}{2}\,,$$

where the notation is defined in Figure 7, α and β are constants which must be determined by the equilibrium conditions. Equalizing to zero the x and y components of the momentum of the forces one obtains the equations

$$\int_{-e/2}^{e/2}\int_{-L/2}^{L/2} x\,\sigma(x,y)\,dxdy=\frac{T}{\sqrt{2}}D\cos\theta$$

$$\int_{-e/2}^{e/2}\int_{-L/2}^{L/2} y\,\sigma(x,y)\,dxdy=\frac{T}{\sqrt{2}}D\sin\theta\,.$$

Solving the integrals after replacing the explicit expression for $\sigma(x,y)$, two equations for the coefficients α and β are obtained, which give the solutions

$$\alpha=\frac{12\,TD}{\sqrt{2}L^3e}\cos\theta,\qquad \beta=\frac{12TD}{\sqrt{2}Le^3}\sin\theta\,.$$

Inserting this in the original expression for $\sigma(x,y)$ one arrives to

$$\sigma(x,y)=\frac{T}{\sqrt{2}Le}+\frac{12TD}{\sqrt{2}Le}\left(\frac{x}{L^2}\cos\theta+\frac{y}{e^2}\sin\theta\right),\qquad \frac{L}{2}\leq x\leq\frac{L}{2}\,,\qquad \frac{e}{2}\leq y\leq\frac{e}{2}.$$

This equation indicates that the maximal stresses occur in the points

$$(x,y)=\left(\pm\frac{L}{2}\,,\pm\frac{e}{2}\right),$$

therefore

$$\sigma_{\text{Maximal}}=\frac{T}{\sqrt{2}Le}+\frac{6TD}{\sqrt{2}Le}\left(\frac{1}{L}\cos\theta+\frac{1}{e}\sin\theta\right),$$

or

$$T=\frac{\sqrt{2}Le\sigma}{1+\frac{6D}{L}\cos\theta+\frac{6D}{e}\sin\theta}\,.$$

Here $\sigma=\sigma_{\text{Maximal}}$ is the tensile or compressive normal stress in any of the critical points $(\pm L/2\ ,\pm e/2)$ in the pad plate fixed to the concrete wharf face.

However the base of the eye plate, welded to the pad, also experiences a shear stress

$$\tau=\frac{T}{\sqrt{2}Le}$$

And the condition that fluency does not occurs is

$$\frac{\sigma_Y}{2}>\sqrt{\left(\frac{\sigma}{2}\right)^2+\tau^2}\,.$$

Combining these equations, the condition for the mechanical stability becomes

(+569) 6251 6487
mlagos@ecofender.cl

$$T < \frac{\sqrt{2}Le\sigma_Y}{2\sqrt{1+\frac{1}{4}\left(1+\frac{6D}{L}\cos\theta+\frac{6D}{e}\sin\theta\right)^2}}.$$

Table 1 follows from setting $L = 0.15$ m, $e = 0.05$ m, $D = 0.075$ m, and from assuming that the material is steel A36. Steel A36 is know this way because $\sigma_Y =$ 36 ksi =250 MPa and a list of its characteristics is in the figure at the end of this section.

Table 1

Limiting chain tensions to prevent failure in the eye plate and pad.

$\theta°$	T [kN] (deformación)
0	593
10	452
20	369
30	318
45	273.5

Stresses in the shackle

Chain tension T is related with the normal stresses in the two arms of the shackle and chain rings, represented in Figure 8 with magnitudes expressed in millimeters, by

$$T = 2\pi \times 0.020^2 \times \sigma \quad \text{(shackle and chain)}\,,$$

where T turns out in kN when σ is expressed in kPa. For the load T and shear stress τ in the bolt of the shackle it holds

$$T = \pi \times 0.020^2 \times \tau \quad \text{(perno del grillete)}\,.$$

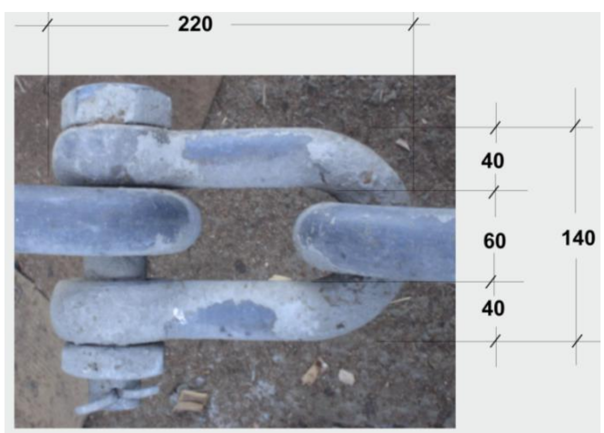

Figure 8

Replacing the critical values of steel A36 the maximal loads these parts can resist. Values are given in Table 2.

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mlagos@ecofender.cl

Table 2
Limiting tension of chain to avoid shackle and bolt failure.

Part	T [kN] (deformation)
Shackle	628
Shackle bolt	314

Deformation of pad plate by a transversal load over the anchoring ensemble

To ensure the pad plate will undergo no deformation by the torsion exerted by the eye plate when transversally loaded ($\theta \neq 0$), the local curvature radius ρ of the pad in the union with the transversally loaded eye plate is calculated next.
The eye plate exerts on the pad plate a moment $M = TD\,\sin\theta$ around of the x axis, that is the axis running along the greatest dimension L of the pad, in the common section of both plates, the eye and pad ones. According to the Euler–Bernoulli equation,

$$\frac{1}{\rho} = \frac{f''}{(1+f')^{3/2}} = \frac{M(y)}{EI},$$

where ρ is the local curvature radius of a plate or beam extending along the y axis, $f(y)$ is the deflection at each point y, $M(y)$ is the flexing moment at point y, E is the Young modulus of the material and I the moment of inertia around an axis passing by the center of mass of the transversal section. Particularizing to our case one obtains that

$$\frac{1}{\rho} = \frac{12TD\,\sin\theta}{ELd^3},$$

where $d = 1.25" = 0.0318\,\mathrm{m}$ represents the ploate thickness. Replacing the value $E = 200\,\mathrm{GPa}$ of steel A36, and assuming that $T\,\sin\theta = 200\,\mathrm{kN}$, which is a magnitude greater than what may be expected in a real situation, the equation gives

$$\rho = 20.8\,\mathrm{m}\,.$$

The curvature radius is then very big, which means that the plate of thickness 1.25" conserves practically flat in the most critical region, which is the union with the eye plate. The elastic deformation there is negligible.

Shielding against corrosion of the metallic parts

Galvanization in hot of type G20. Coating with a zinc shielding of 3.5 μm thick to all metallic parts. This protection agains rusting is recommended for agresive media, like the marine one.

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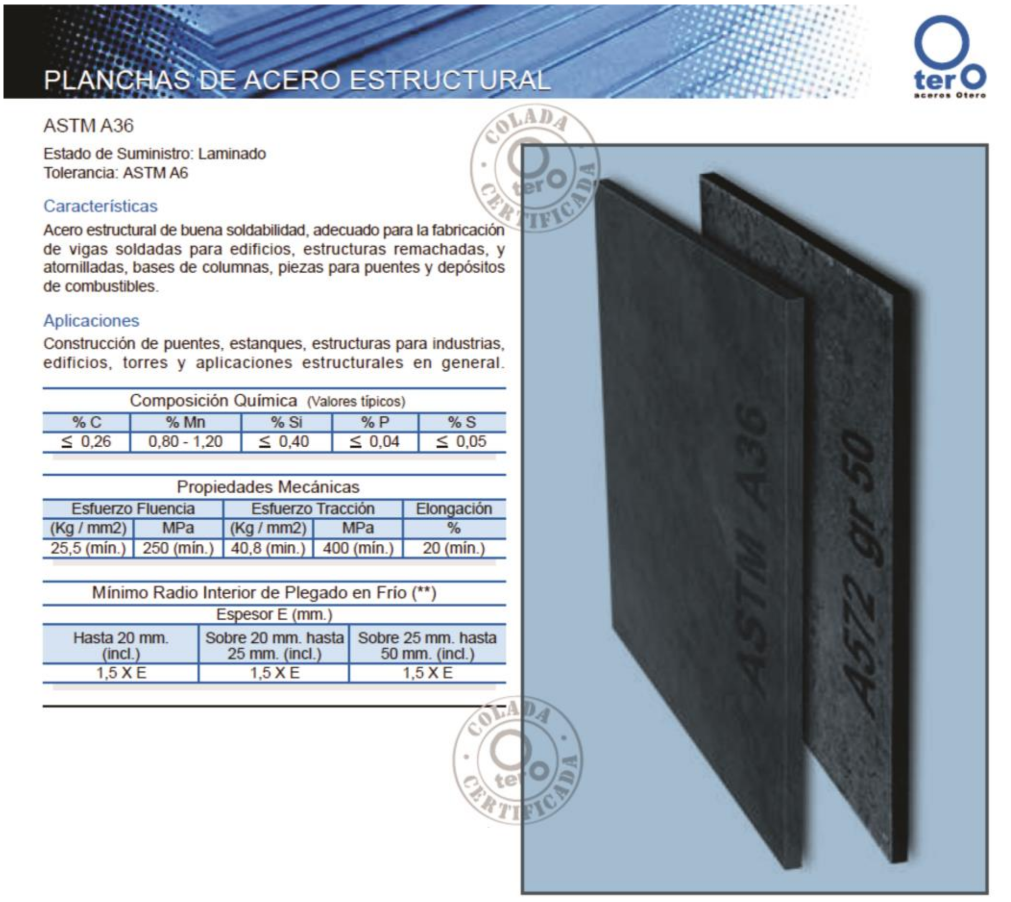
Steel A36


PLANCHAS DE ACERO ESTRUCTURAL

ASTM A36

Estado de Suministro: Laminado
Tolerancia: ASTM A6

Características

Acero estructural de buena soldabilidad, adecuado para la fabricación de vigas soldadas para edificios, estructuras remachadas, y atornilladas, bases de columnas, piezas para puentes y depósitos de combustibles.

Aplicaciones

Construcción de puentes, estanques, estructuras para industrias, edificios, torres y aplicaciones estructurales en general.

Composición Química (Valores típicos)

% C	% Mn	% Si	% P	% S
≤ 0,26	0,80 - 1,20	≤ 0,40	≤ 0,04	≤ 0,05

Propiedades Mecánicas

Esfuerzo Fluencia		Esfuerzo Tracción		Elongación
(Kg / mm2)	MPa	(Kg / mm2)	MPa	%
25,5 (mín.)	250 (mín.)	40,8 (mín.)	400 (mín.)	20 (mín.)

Mínimo Radio Interior de Plegado en Frío (**)

Espesor E (mm.)		
Hasta 20 mm. (incl.)	Sobre 20 mm. hasta 25 mm. (incl.)	Sobre 25 mm. hasta 50 mm. (incl.)
1,5 X E	1,5 X E	1,5 X E

2. Calculation of the internal constituents and unions

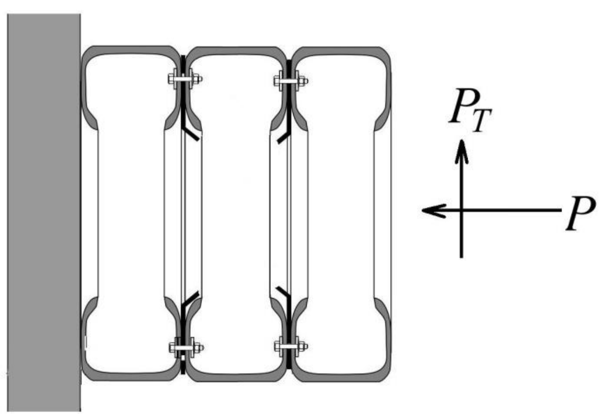


Figure 9

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mlagos@ecofender.cl

Normal forces, perpendicular to the wharf berthing side, do not produce significant stresses on the elements which hold the tires together. The transversal force P_T is the one making the metallic parts to work. Force P_T distributes evenly in the 12 kidney shaped and the corresponding 12 bolts. The calculation assumes the extreme situation in which the fender is in its maximum compression, exerting the maximal reaction force P, and the vessel displaces parallel to the wharf vertical face making the hull side to slide over the rubber surface of the fender. Thus we have

$$P_T = \mu P,$$

Table 4.4.1. Hull Pressure Guide

Type of vessel	Hull Pressure kN/m²
Container vessels 1st and 2nd generation	<400
3rd generation (Panamax)	<300
4th generation	<250
5th and 6th generation (Superpost Panamax)	<200
General cargo vessels	
=/< 20.000 DWT	400-700
> 20.000 DWT 40	<400
Oil tankers	
=/< 60.000 DWT	<300
> 60.000 DWT	<350
VLCC	150-200
Gas carriers (LNG /LPG)	<200
Bulk carriers	<200
SWATH	}
RO-RO vessels	} these vessels are usually belted
Passenger Vessels	}

where P is the specified maximal reaction force, which depends on the hull resistance. We consider here a design vessel of maximal size established by the **Panamax** norm. According to the inform of 2002 of PIANC, one of its tables is reproduced at the end of this paragraph, the recommended maximal pressure on the hull of these vessels is $300\ \mathrm{kN/m^2}$. The area of an Ecofender 2300 fender is $4.9\ \mathrm{m^2}$, hence the maximal force per fender is

$$P = 300\ \mathrm{kN/m^2} \times 4.9\ \mathrm{m^2} = 1470\ \mathrm{kN}.$$

Considerando ahora $\mu = 0.7$,

$$P_T = 1029\ \mathrm{kN}\ \ (= 105.0\ \mathrm{ton–f}).$$

Bolts in between the tires

The coupling of the tires is not entirely rigid because because the three steel plates, the central disc and the two kidney shaped plates have in between the rubber side walls of the tires, which are the ones exerting the forces, as shown in Figure 10.

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mlagos@ecofender.cl

The rubber parts do not exert forces on the bolts, but over the central disk and the two kidned shape plates. The friction between the steel constituents and the rubber surfaces is what support the forces and distributes them over the steel surfaces in order to reduce the stress on the rubber parts. The force exerted by the bolts on the rubber surfaces by just their tightening at mounting must be comparable to P_T, in order to produce a friction force which should not be surpassed in the situation of extreme demand. Each bolt then supports a permanent normal stress σ and a shear stress τ_1 when the fender is subjected to a shear force.

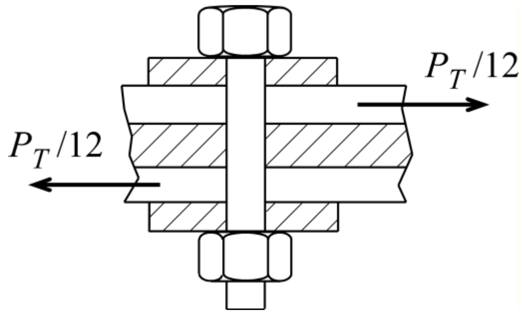


Figure 10

The tightening of the bolts at mounting is such that in the critical situation both stresses be of similar magnitude. Then we have a situation of combined normal and shear stresses satisfying

$$\sigma = \tau_1 = \frac{P_T/12}{\pi d_1^2/4}.$$

Plastic yield is prevented if

$$\sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau_1^2} < \frac{\sigma_Y}{2}.$$

Combining the equations it turns out

$$d_1 > \sqrt{\sqrt{\frac{5}{2}}\frac{P_T}{3\pi\sigma_Y}}.$$

Steel volts available in the market are classified in different grades attending to the yield stresses σ_Y of the material they are made. In millimetric bolts the grade is identified by a number at the head, indicating the approximate ultimate stress of the steel. In the norm in inches the grade is indicated by radial marks in the bolt head. Appendix B shows a table of the bolt grades with their mechanical characteristics. It is shown there that weaker steel employed in bolts (bolts of grade 2) have an effective yield stress close to $\sigma_Y = 33$ ksi $=$ 228 MPa, that is a resistance close to that of the steel A36. Bolts of higher quality (grade

(+569) 6251 6487
mlagos@ecofender.cl

8.2 or class 12.9) reach to yield stresses of more than 3 or 4 times the ones of bolts of lower class or grade.

In order to assume the less favourable situation the equations written above are evaluated replacing the modest value $\sigma_Y = 250$ MPa (reference steel A36). It is obtained that the bolt diameter d_1 musbe such that

$$d_1 > 0.0263 \text{ m} \qquad \text{(minimal diameter coupling bolts grade 2 SAE).}$$

The 1” bolts ($d_1 = 0.0254$ m) considered in the design of the Ecofender 2300 fender would be then slightly sub-dimensioned if lower quality bolts were employed. However, more reasonable standard bolts of grade 5 SAE have a minimal yield stress of 510 – 586 MPa (74 – 85 ksi), which gives for the bolt diameter

$$d_1 > 0.0172 - 0.0184 \text{ m} \quad \text{(coupling bolts grade 5 SAE).}$$

1” bolts of this grade provide a safety factor of at least 1.38. Bolt tightening torques are given in the table below

Nominal diameter	Minimal pretension kips	
Inches	ASTM A325	ASTM A490
1/2	11	56
5/8	19	24
3/4	28	35
7/8	39	49
1	51	64
1 1/8	56	80
1 1/4	71	102
1 3/8	85	121
1 1/2	103	148

Elastic deformation of the kidney shaped plates

The kidney shaped plates are asimilable to rectangular plates (Figure 11). The rubber surface applies a uniform distribution of forces on them, particularly for a small overall bending of the plate. The purpose of the next calculation is to dimension this small overall bending.

Euler–Bernoulli equation:

$$\frac{y''}{(1+y'^2)^{3/2}} = \frac{M(x)}{EI},$$

(+569) 6251 6487
mlagos@ecofender.cl

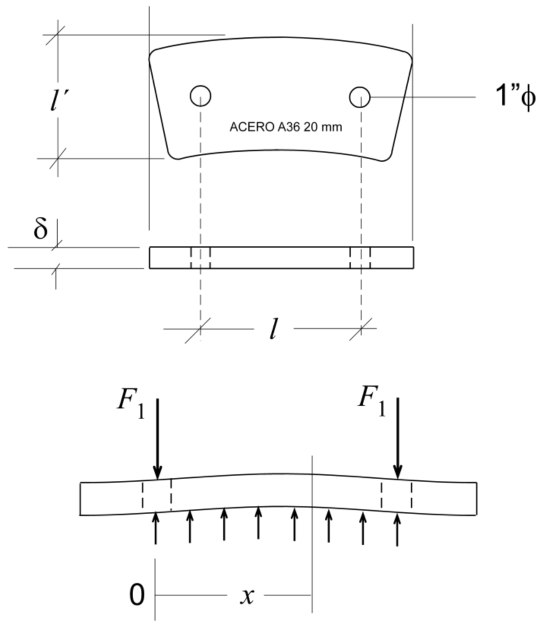


Figure 11

where $M(x)$ is the flexing moment at point x, and $y(x)$ is the corresponding deflection, E is the Young modulus of steel and δ the plate thickness. As $y' \ll 1$ we can write

$$y'' = \frac{M(x)}{EI},$$

and replacing $M(x)$

$$EIy'' = -F_1 x + \int_0^x (x - x') \frac{2F_1}{ll'} l' dx' = -F_1 x + \frac{F_1}{l} x^2.$$

Integrating two times with the conditions

$$y'\left(\frac{l}{2}\right) = 0, \;\; y(0) = 0,$$

it is obtained that

$$y(x) = \frac{F_1}{6EI}\left(-x^3 + \frac{x^4}{2l} + \frac{l^2}{2}x\right), \quad \text{with} \quad I = \frac{\delta^3 l'}{12},$$

Hence the maximum deflection takes place at the central point and turns out to be

$$y\left(\frac{l}{2}\right) = \frac{5F_1 l^3}{16E\delta^3 l'}.$$

Substituting $l = 190$ mm, $l' = 150$ mm, $\delta = 20$ mm, $F_1 = \frac{1029 \text{ kN}}{12} = 85.8$ kN, $E = 2.0 \times 10^{11}$ Pa (steel A36), we obtain

$$y\left(\frac{l}{2}\right) = 0.766 \text{ mm} \quad \text{(maximum deflection)}.$$

Then the thickness 20 mm for the kydney shape plates is right. There is no significant deformation even for a considerable bolt tightening.

(+569) 6251 6487
mlagos@ecofender.cl

Internal chains

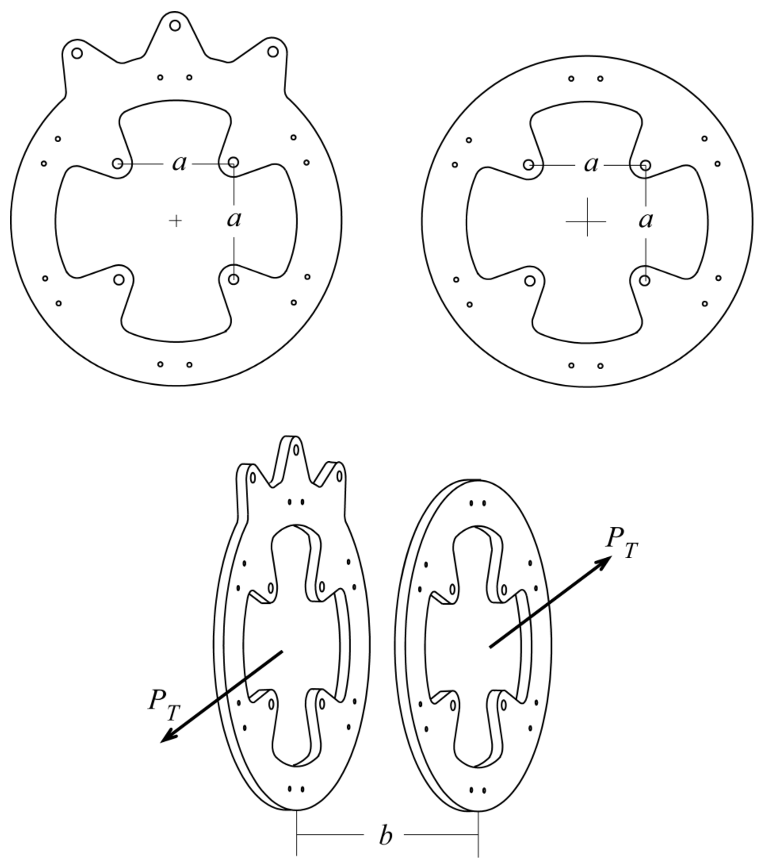


Figure 12.
$a = 778$ mm, $b = 750$ mm

T is the tension of each chain, T' is the magnitude of the vector sum of the tensions. The angle between the two chains is 2ϕ in the plane they define. The geometry is shown in figures 12 and 13, the chains follow the diagonals of the parallelepiped.

From the geometry of the forces (Figure 13) we have

$$T' = \frac{P_T}{\cos\theta} \qquad \cos\theta = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\frac{1}{2}T' = T\cos\phi$$

$$T = \frac{P_T}{2\cos\phi\cos\theta}$$

(+569) 6251 6487
mlagos@ecofender.cl

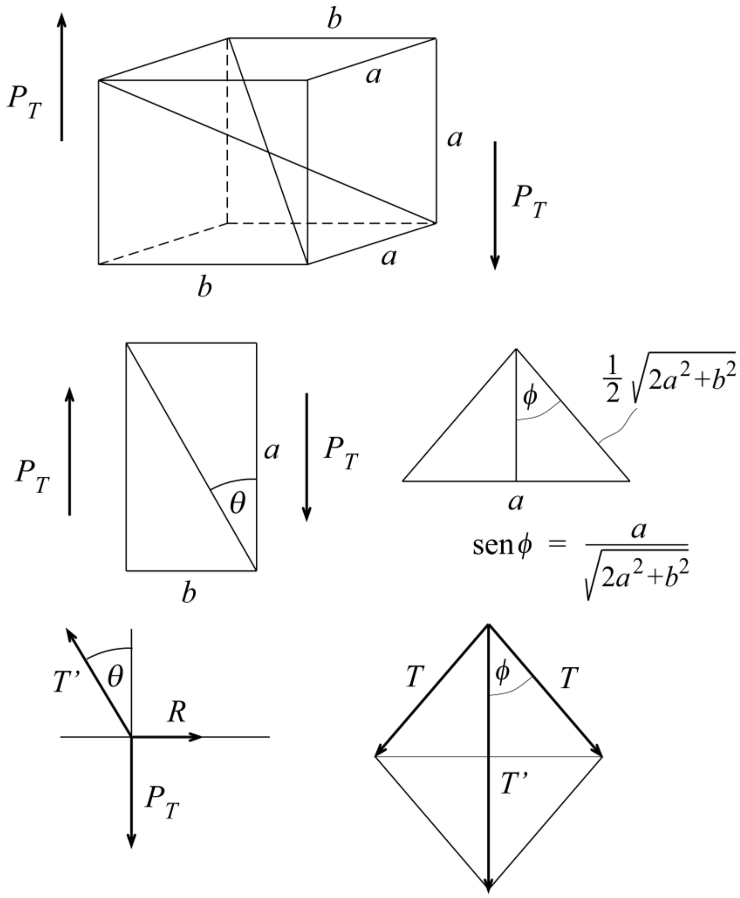


Figure 13

$$\cos\phi = \sqrt{1 - \frac{a^2}{2a^2 + b^2}} = \sqrt{\frac{a^2 + b^2}{2a^2 + b^2}}$$

$$\cos\theta \cos\phi = \frac{a}{\sqrt{a^2 + b^2}} \sqrt{\frac{a^2 + b^2}{2a^2 + b^2}} = \frac{a}{\sqrt{2a^2 + b^2}}$$

and the tension at each chain is

$$T = \frac{\sqrt{2a^2 + b^2}}{2a} P_T.$$

Replacing values the tension turns out to be

$$T = 881 \text{ kN} = 89.9 \text{ ton–f}.$$

The ultimate stress of the chain considered in the design is $T_{\text{máx}} = 46.5$ ton–f =456 kN, according to the specifications of the manufacturer.

(+569) 6251 6487
mlagos@ecofender.cl

According to the previous calculation, the internal chains may should fail for a value $P_T = 1029$ kN of the transversal force applied to the fender. This force assumes the maximal demand that can be applied to the fender. Next the value of P_T at which the rupture takes place is calculated:

$$P_T(\text{máx.}) = \frac{2a}{\sqrt{2a^2 + b^2}} T_{\text{máx}} \,.$$

Substituting the specified magnitude $T_{\text{máx}} = 456$ kN we have

$$P_T(\text{máx.}) = 533 \text{ kN}.$$

The internal chains constitute a warning for excessive transversal load. The force $P_T = 1029$ kN, which is an extreme condition, assumes that the vessel presses a single fender (the critical fender) to its maximum, while displaces parallel to the wharf side sliding over the fender, with a friction coefficient of 0.7.

Shackle of internal chains

According to the design of the Ecofender 2300 fender, the shackles of the internal chains have a larger ultimate stress than the chains.

Disks

The disks work frontally to compression. The critical points are at the eyes, which must support the weigth of the fender (5.9 tonnes with cumulated water), which constitutes a comparatively modest load.

Dr. Miguel Lagos Infante
Professor, Faculty of Engineering
University of Talca, Chile

(+569) 6251 6487
mlagos@ecofender.cl

Apendix A
Criterion for mechanical failure

The criterion for determining the load at which the material subjected to a strain field yields and start plastical deformation, or initiates fracture, is not complex in a bidimensional regime. Combining the equations for the normal stresses in the directions of the axes x and y,

$$\sigma_x = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta \,,$$

$$\sigma_y = \sigma_1 \sin^2\theta + \sigma_2 \cos^2\theta \,,$$

and the corresponding shear stress

$$\tau_{xy} = (-\sigma_1 + \sigma_2)\, \sin\theta\, \cos\theta \,,$$

as functions of the stresses σ_1 and σ_2 in the principal directions 1 y 2, and the angle θ which makes the axis x with the principal direction 1, the invariance of magnitudes

$$p = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2}$$

and

$$k = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2}$$

is shown. The first invariant is the hydrostatic pressure, and the meaning of the second is seen next. It is straightforward that

$$\frac{d\sigma_x}{d\theta} = -\frac{d\sigma_y}{d\theta} = \tau_{xy} \,,$$

which means that the normal stresses σ_x y σ_y assume extremal values in the principal directions and the shear stress vanishes. To fix ideas write

$$\sigma_{\max} = \sigma_1, \qquad \sigma_{\min} = \sigma_2 \,.$$

It is clear that the critical condition for the mechanical stability of the material is connected with the maximal stress σ_1. However, to write the critical condition is necessary to recall that an isotropic hydrostatic pressure cannot produce any flow of the material. The deformation then occurs when the deviation of the normal stress with respect to the hydrostatic pressure, in the direction in which the former is maximal, exceedes a limit imposed by the nature of the material. The condition for the material does not flow is thus

$$\sigma_1 - p < \sigma_Y - \frac{\sigma_Y + 0}{2} = \frac{\sigma_Y}{2},$$

(+569) 6251 6487
mlagos@ecofender.cl

where σ_Y is the yield stress of the material in a condition of uniaxial tension. In the left side of the inequality the hydrostatic pressure was substracted. By the definition of p,

$$\sigma_1 - p = \frac{\sigma_1 - \sigma_2}{2}$$

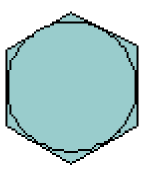
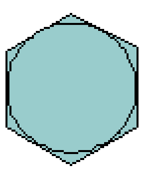
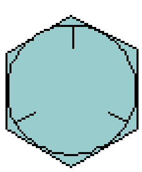
which is just the invariant k, which shows that the condition for the mechanical stability of the system, written in an arbitrary frame of reference

$$\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} < \frac{\sigma_Y}{2},$$

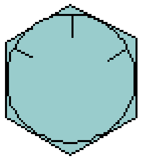
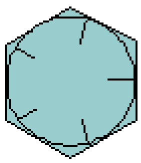
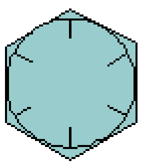
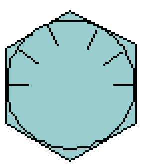
does not depend on the choice of the frame of reference.

Apendix B
Grade SAE of bolts, norm in inches and millimeters

Inches, norm USA

SAE Grade	Head Marking	Size Range (in)	Minimum Proof Strength (ksi)	Minimum Tensile Strength (ksi)	Material
1		1/4 - 1 1/2			Low - medium carbon steel
2		1/4 - 3/4 7/8 - 1 1/2	55 33	74 60	Low - medium carbon steel
5		1/4 - 1 1 1/8 - 1 1/2	85 74	120 105	Medium carbon steel, Q & T

(+569) 6251 6487
mlagos@ecofender.cl

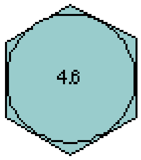
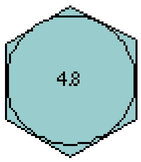
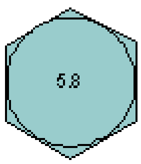
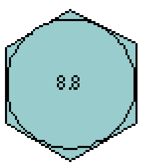
SAE Grade	Head Marking	Size Range (in)	Minimum Proof Strength (ksi)	Minimum Tensile Strength (ksi)	Material
5.2		1/4 - 1	85	120	Low carbon martensite steel, Q & T
7		1/4 - 1 1/2	105	133	Medium carbon alloy steel, Q & T
8		1/4 - 1 1/2	120	150	Medium carbon alloy steel, Q & T
8.2		1/4 - 1	120	150	Low carbon martensite steel, Q & T

$$1 \text{ ksi } (1000 \text{ lb/in}^2) = 6.8948 \text{ MPa } (10^6 \text{N/m}^2)$$

Millimeters

Class	Head Marking	Size Range *(mm)*	Minimum Proof Strength *(10^6 Pa)*	Minimum Tensile Strength *(10^6 Pa)*	Material

(+569) 6251 6487
mlagos@ecofender.cl

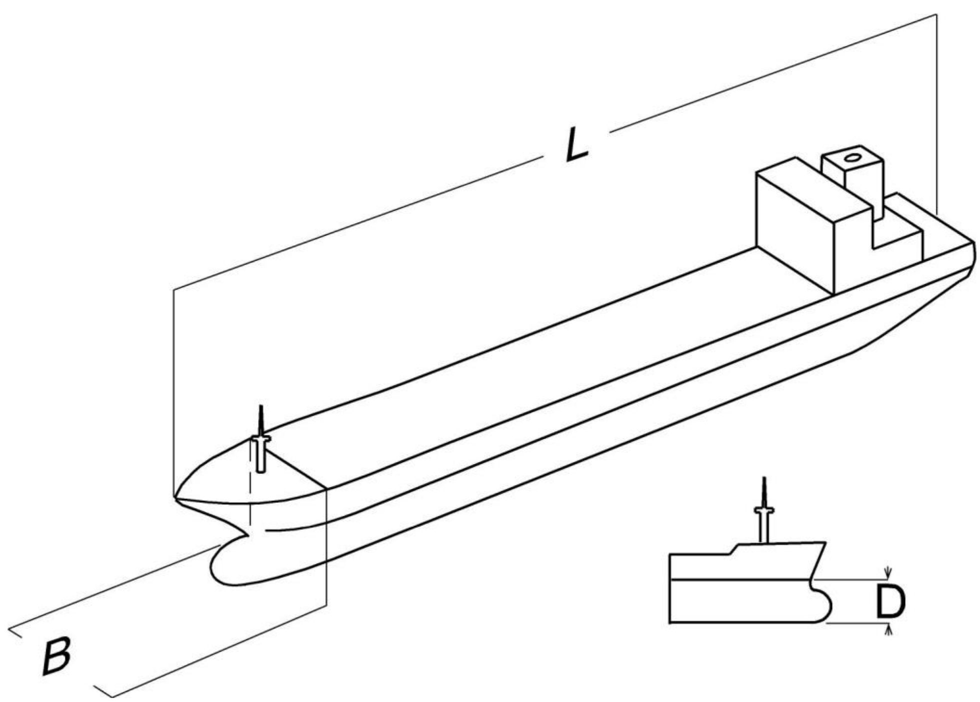
Class	Head Marking	Size Range *(mm)*	Minimum Proof Strength *(10^6 Pa)*	Minimum Tensile Strength *(10^6 Pa)*	Material
4.6		M5 - M36	225	400	Low or medium carbon steel
4.8		M1.6 - M16	310	420	Low or medium carbon steel
5.8		M5 - M24	380	520	Low or medium carbon steel
8.8		M16 - M36	600	830	Medium carbon steel, Q & T

(+569) 6251 6487
mlagos@ecofender.cl

Berthing energy calculation

Design vessel: Panamax

Type and capacity	Container vessel, up to 5000 TEU
Length (L)	294 m
Breadth (B)	32.2 m
Draft (D)	12.0 m
Displacement (M)	75690 ton
Block coefficient (C_B)	0.65
Maximal hull pressure	300 $\mathrm{kN/m^2}$



(+569) 6251 6487
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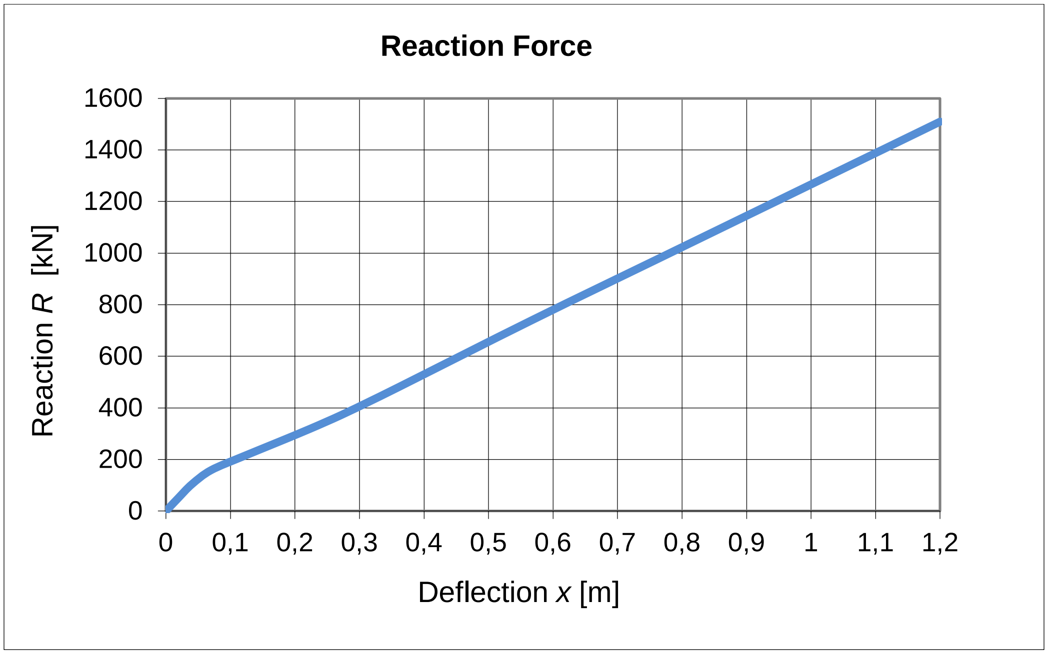
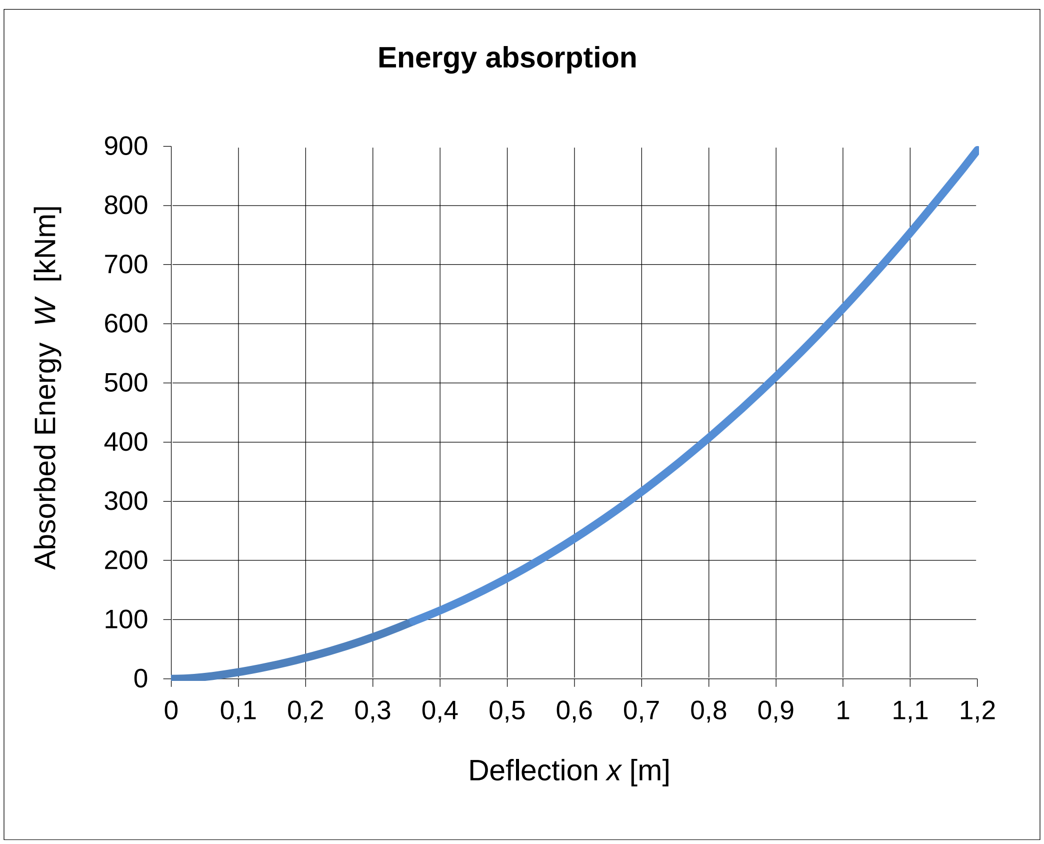
Fender

IDIEM - Informe de ensayo No 795525 - 13/12/2012		
Ecofender 2300 - 3 tires		
Deflection x [m]	Reaction P [kN]	Stored Energy W [kJ]
0	0	0
0,0189	50	0,474
0,0402	104	2,112
0,0696	158	5,964
0,1188	211	15,042
0,1716	265	27,609
0,2235	319	42,762
0,2715	372	59,346
0,3165	426	77,31
0,3603	480	97,143
0,567	739,7	215,76
0,762	976,8	371,31
0,79	1010,9	397,7
0,81	1035,2	417,1
0,85	1083,9	447,2
0,9	1144,7	510,7
0,95	1205,5	566,9
1	1266,3	626,2
1,05	1327,2	688,5
1,1	1388,0	753,9
1,167	1469,5	846,2
1,2	1509,6	893,7

...... Situation corresponding to a pressure 300 kN/m^2 on hull.

(+569) 6251 6487
mlagos@ecofender.cl





MAGELLAN
CONSULTING GROUP
MARINE SOLUTIONS
(+569) 6251 6487
mlagos@ecofender.cl
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BERTHING ENERGY CALCULATION

DATA

Vessel

Initial speed	$v_0 =$	0.103	m/s	(0.2 knot)
Displacement	$M =$	75700	ton	
Length	$L =$	294	m	
Breadth	$B =$	32.2	m	
Draft	$D =$	12	m	
Flat length	$L' =$	100	m	

Fender

Energy absorption	$W =$	846	kN m
for a reaction	$R =$	1470	kN
Fender area	$\Omega =$	4.9	m^2
Maximal deflection	$x_M =$	1.17	m

CALCULATIONS

Auxiliary parameters:

Block coeff.	$C_b =$	0.650		
Giration radius	$K =$	68.65	m	(Formula PIANC)

Coefficients:

Excentricity	$C_e =$	0.466	(Formula PIANC)
Virtual mass	$C_m =$	1.900	(Formula Hueda)
Cushion effect	$C_c =$	1	
Elasticity	$C_s =$	1	

Energies:

Kinetic energy vessel	$E = (1/2)Mv_0^2 =$	401.6	kN m
Non parallel berthing	$E' = C_e C_m C_c C_s E =$	355.6	kN m
Parallel berthing	$E'' = C_m C_c C_s E =$	763.1	kN m

(+569) 6251 6487
mlagos@ecofender.cl

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CONCLUSIONS

Pressure of critical fender on hull	$R/\Omega =$	300.0	$\mathrm{kN/m^2}$
Energy excess on critical fender	$(E' - W)/W$	-58.0	%
Minimal number of fenders	$n =$	1.0	defensas
Safe distance between fenders	$d \leq$	11.13	m

When the pressure over hull reaches $300\ \mathrm{kN/m^2}$ the critical fender (the first fender making contact with the vessel in berthing), has stopped the point of contact of the vessel, which initiated a rotational motion around the fender. The critical fender conserves 58% of its capacity after completing its function of aligning the vessel with the fender line.

Berthing in abnormal conditions

PIANC recommends inclusion of a coefficient between 1.1 and 2.0 to take into account the possibility of berthing in conditions whose parameters are out of the ranges which are considered as normal in the calculations. The coefficient should take into account the costs of a possible accident when these have a special character, for example, when operating with flammable, explosive or contaminant freights. In our case a coefficient as big as the maximum recommended by PIANC (2.0) gives a berthing energy smaller than the maximum energy the fender Ecofender 2300 can absorb without exceeding the pressure $300\ \mathrm{kN/m^2}$ over the hull.

Miguel Lagos Infante, PhD
Professor, Faculty of Engineering
University of Talca, Chile

(+569) 6251 6487
mlagos@ecofender.cl